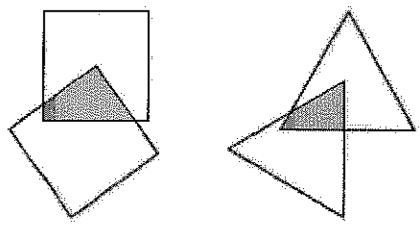
2011 John O'Bryan Mathematical Competition
Junior-Senior 5-person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper for each numbered problem**. Place your team letter in the upper right corner of each page that will be turned in (failure to do this will result in no score). Problems are equally weighted; **teams must show complete solutions (not** [redacted]

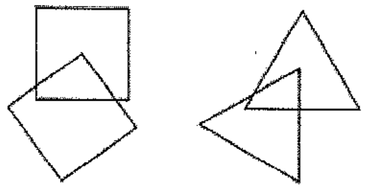
1. Two identical squares overlap with the corner of one square at the center of the other.

    a. Find the proportion of the area of each square that is in the overlapped section.
    b. Prove that the proportion of area in part (a) is independent of the relative positions of the two squares.
    c. If we replace the squares by identical equilateral triangles, with the corner of one at the center of the other, is the overlap still independent of relative positioning? Explain!

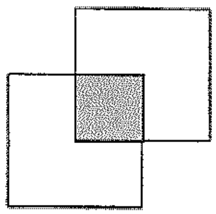


2. Two railroad cars (labeled A and B) are in a [redacted]

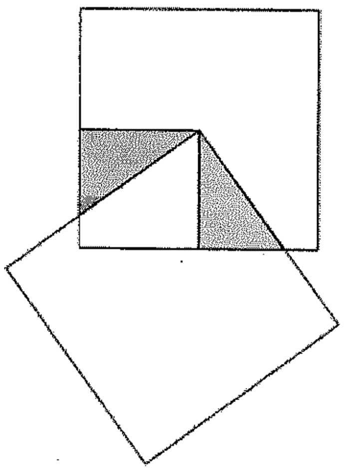
1. Two identical squares overlap with the corner of one square at the
a. Find the proportion of the area of each square that is in the overl
b. Prove that the proportion of area in part a is independent of the
c. If we replace the two word squares by identical equalateral trian



a. By rotating the lower rectangle we see the area is obviously 1/4 of t

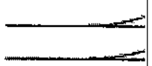


b. In the following picture the two filled triangles have the same area,



c. No, the overlap now changes area. This is easiest seen by looking a
connected all such points with straight lines to form nine congruent tri

2. Two railroad car
Either end of the



1 Use the engine to
2. Leaving car A wh
3. Unhitch car B, m
4. Hitch to car B, p
5. Pull both cars ba
6. Unhitch car A (le
7. Drive the engine
8. Pull car A down t
9. Return the engine

3. T

a. Jus

{1,

Every

b. Let

First

F_{3k+1}

Note:

c. It

d. An

$F_{4(k+}$

4. As a d

We know
get

when

5. Let a and b be two

If we look at the units

If we sum these possib

6. What

Since 15

So there

7. If $\sin x + \cos x$

$\frac{1}{4} = (\sin x + \cos x)^2$

We also have that

So, $\sin^4 x + \cos^4$